Maths – KS4 Higher

Fact Sheets:

- Number, Ratio and Proportion
- Algebra
- Geometry and Measures
- Probability and Statistics



Number Ratio and Proportion - Higher

, o
Estimate
Round each value to one significant figure
Standard form

 $a \times 10^{n}$, where $1 < a < 10^{n}$

Reciprocal

Reciprocal of 7 is $\frac{1}{7}$, reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ etc

Sequences

Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21

Geometric Sequence: each term is multiplied but he same constant to get the next number.

E.g. 3, 12, 48, 191, (x by 4 each time)

Recurring Decimals

Form two equations where the digits following the decimal point are the same, and therefore can be cancelled

Upper and lower bounds

Look at the value above and below for the same place value. LB and UB will be half way between these points

e.g. 17 rounded to the nearest integer



e.g. 24.6 roudned to one decimal place.

Simplifying Surds

Find a factor that is a square number

$$\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$$

Manipulating surds

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Rationalising Surds

Rationalise by removing any surds from the denominator

E.G with surd.

$$\frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{3} \times 5}{\sqrt{5} \times 5} = \frac{2\sqrt{15}}{\sqrt{25}} = \frac{2\sqrt{15}}{5}$$

E.G with surd expressions multiply by top and bottom by the denominator with the opposite sign.

$$\frac{5}{3+\sqrt{2}} = \frac{5 \times (3-\sqrt{2})}{(3+\sqrt{2}) \times (3-\sqrt{2})} = \frac{5(3-\sqrt{2})}{9-\sqrt{4}}$$
$$= \frac{5(3-\sqrt{2})}{7}$$

Fractions

Add and Subtract – ensure the fractions have the same denominator before adding numerators

$$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$

Multiply – multiply numerators and denominators

$$\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

Divide – take reciprocal of the second fraction and then multiply the new numerators and denominators

$$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$$

Percentages

Finding percentages of an amount

Multipliers:

To find the multiplier for a percentage, divide by 100

Use multipliers on a calculator paper e.g. 35% of 370 = 0.35 x 370

Increasing and decreasing a given amount

Calculator:

 $Orginal\ Amount\ x\ mutiplier = new\ amount$

Non-calculator: find the increase or decrease and add to the original amount

Finding percentage increase or decrease (profit/loss)

$$\frac{value\ of\ increase/decrease}{Original} \times 100$$

Writing an amount as a percentage of the original

$$\frac{Amount}{Original} \times 100$$

Reverse Percentage – finding the original amount

$$Orginal\ Amount = \frac{New\ Amount}{multiplier}$$

		·
Growth & Decay / Compound interest	Dividing by decimals:	Conversions
	Write the calculation as a fraction	10 millimetres = 1 centimetre 15 minutes = 0.25
original amount $ imes$ multiplier time	Form an equivalent fraction to makes integers	hours
	(multiply by powers of 10)	100 centimetres = 1 metre 30 minutes = 0.5
Where the multiplier is the percentage, increase or	3. Use short division (bus stop) to calculate	hours
decrease from 100%, converted to a decimal.		1000 metres = 1 kilometre 45 minutes = 0.75
e.g.	e.g. $460 \div 0.4 = \frac{460}{0.4} = \frac{4600}{4} = 1150$	hours
30% decrease is 70% = 0.7	0.4 4	1000cm ³ = 1 litre 1000g = 1 kilogram
30% increase is 130% = 1.3		1000ml = 1 litre
Compound Units (rearrange as necessary)	Error Intervals	Negative numbers
	least possible value $\leq x <$ greatest possible value	Adding and subtracting: (vertical number lines help)
Distance		-3 – 5 = -8
$Speed = \frac{Distance}{Time}$	e.g. A fence is 30 m long to the nearest 10 m.	-3 + 5 = 2
Time	$25 \text{ m} \le l < 35 \text{ m}$	-3 5 = -3 + 5 = 2
		-3 - + 5 = -3 - 5 = -8
$Area = \frac{Force}{Pressure}$	Truncation	-3 + - 5 = -3 – 5 = -8
Pressure	Truncation is a method of approximating a decimal number	
	by dropping all decimal places past a certain point without	Multiplying and dividing:
	rounding.	Different signs – answer will be negative
$Density = \frac{Mass}{Volume}$	e.g. Truncate 3.14159265 to 4 decimal places.	+ x - = -, - x + = -
Volume	= 3.1415	Same signs – answer will be positive
	- 5.1415	- X - = +
Product rule	Order of operations	Rounding to significant figures
If there are m ways to do one thing and n ways to do	B racket	Start from the first non-zero number and round as
another, then there are $m \times n$ ways to do both	Indices	normal, but ensure the place value is correct
	D ivision and M ultiplication	e.g. 345,635 to 2SF = 350,000
	Addition and Subtraction	0.0060821 to 3SF = 0.0608
Index Laws		M of 90 and 120 (Factor Tree & Venn Diagram)
$a^n \times a^m = a^{n+m}$	HCF is the p	roduct of common factors

$$a^{n} \times a^{m} = a^{n+m}$$

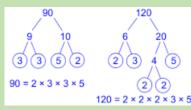
$$a^{n} \div a^{m} = a^{n-m}$$

$$(a^{n})^{m} = a^{nm}$$

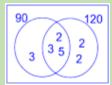
$$a^{0} = 1$$

$$a^{-n} = \frac{1}{a^{n}}$$

$$a^{m} = \sqrt[m]{a^{n}}$$



LCM is the product of common factors and remaining factors.



HCF: 2x3x5 LCM: 2³x3²x5

Number Ratio and Proportion - Foundation

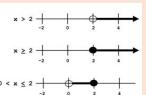
Estimate	Recurring Decimals	Percentages
Round each value to	To change a recurring decimal into a fraction you	
0. 1.16		Finding percentages of an amount
Standard form		1% ÷
$a \times \underline{\hspace{1cm}}^n$, where $1 \le a < 10$		5% ÷ 20% ÷
Reciprocal	Upper and lower bounds	20% ÷ 25% ÷
Reciprocal of 7 is, reciprocal of $\frac{2}{3}$ is etc	Look at the value above and below for the same place	50% ÷
needprocar of 7 isreciprocar of 3 is etc	value. LB and UB will be half way between these points	
Sequences	1	Multipliers:
	e.g. 17 rounded to the nearest integer	To find the multiplier for a percentage, divide by 100
Fibonacci sequence:		
Geometric Sequence:		Use multipliers on a calculator paper
		e.g. 35% of 370 = 0.35 x 370
E.g. 3, 12, 48, 191, (x by 4 each time)	e.g. 24.6 roudned to one decimal place.	
	LB =, UB =,	
Simplifying Surds	Fractions	Increasing and decreasing a given amount
Find a factor that is a number		Calculator:
$\sqrt{96} =$	Add and Subtract – ensure the fractions have the same	= new amount
	before adding the	Non-calculator: find the increase or decrease and add
Manipulating surds	$\frac{4}{5} - \frac{1}{3} =$	to the original amount
$\sqrt{ab} = \sqrt{} \times \sqrt{}$	Multiply — multiply and	3
$\sqrt{\frac{a}{b}} = \frac{\sqrt{}}{\sqrt{}}$	Multiply – multiply and	Finding percentage increase or decrease (profit/loss)
$\sqrt{\frac{1}{b}} = \frac{1}{\sqrt{1 - 1}}$	$\frac{5}{5} \times \frac{3}{3} =$	
		${Original} \times 100$
Rationalising Surds	Divide – takeof the second fraction and	Original
Rationalise by removing any surds from the	thenthe newand	Writing an amount as a percentage of the original
denominator		Amount
E.G with surd.	$\frac{4}{5} \div \frac{1}{3} =$	×
$\frac{2\sqrt{3}}{\sqrt{3}} =$	5 3	
$\frac{1}{\sqrt{5}} =$		Reverse Percentage – finding the original amount
E.G with surd expressions multiply by top and bottom		
by the denominator with the opposite sign.		Orginal Amount =

Growth & Decay / Compound interest	Dividing by decimals:	Conversions
	1.	10 millimetres =
X	2.	100 centimetres =
	3.	30 minutes = hours
Where the multiplier is the percentage, increase or		1000 metres =
decrease from 100%, converted to a decimal.	e.g. 460 ÷ 0.4 =	45 minutes = hours
e.g.		1000cm ³ = 1000g =
30% decrease is 70% =		1000ml = 1000kg =
30% increase is 130% =		
Compound Units (rearrange as necessary)	Error Intervals	Negative numbers
	least possible value $\leq x <$ greatest possible value	Adding and subtracting: (vertical number lines help)
Distance		-3 – 5 =
$Speed = \frac{Distance}{Time}$	e.g. A fence is 30 m long to the nearest 10 m.	-3 + 5 =
1 title	≤ <i>l</i> <	-3 5 =
		-3 - + 5 =
Force	Truncation	-3 + - 5 =
$Area = {Pressure}$	Truncation is	
		Multiplying and dividing:
		Different signs – answer will be
Mass		+ x - =, -x + =
$Density = \frac{Mass}{Volume}$	e.g. Truncate 3.14159265 to 4 decimal places.	Same signs – answer will be
	=	- x - =
Product rule	Order of operations	Rounding to significant figures
If there are m ways to do one thing and n ways to do	В	Start from the first number and round
another, then there are $m \times n$ ways to do both	1	as normal, but ensure the place value is correct
·	D and M	e.g. 345,635 to 2SF =
	A and S	0.0060821 to 3SF =
Index Laws	Prime Factorisation HCF and L	.CM of 90 and 120 (Factor Tree & Venn Diagram)
$a^n \times a^m =$	HCF is the	
$a^n \div a^m =$	90 LCM is the	
$(a^n)^m =$	9 10 6 20	90120
$a^0 =$	2 3 4 5	HCF:
$a^{-n} =$		$\left(\begin{array}{cc} 3 \left(3 & 2 \\ 3 & 5 \end{array}\right) & 2 \end{array}\right)$
$a^{\frac{n}{m}} =$	22	3 2 LCM:
•	==	

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Linear Inequalities



Open circle: </>>

Closed circle: \leq / \geq

Algebriac proof – toolkit

Even numbers: 2n, 2n+2, 2n+4,... Odd numbers: 2n+1, 2n+3, 2n+5,...

Sum: add

Product: multiply Difference: subtract

Show it's a multiple: factorise

Show it's even: show it's a multiple of 2 Show it's odd: show it's a multiple of 2, plus 1

Straight line graphs

$$y = mx + c$$

 $m = gradient$
 $c = y - intercept$

positive gradient

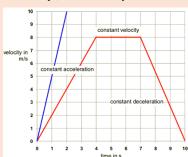
negative gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change \ in \ y}{change \ in \ x}$$

Parallel lines – have equal gradients

Perpendicular lines – If L₁ and L₂ are perpendicular then $m_2 = -\frac{1}{m_1}$

Velocity / Time Graphs



Iteration – showing a root lies between 2 points:

If there is a change in sign for y for two particular values of

x then we can say there is a root between these values of x and we can say that the equation f(x) = 0 will have a solution

Gradient = acceleration

Area = distance travelled

Completing the square

Quadratic expression factorised by completing the square:

$$(x+a)^2+b$$

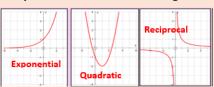
Turning point of graph occurs at (-a, b)

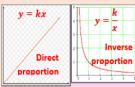
Solve quadratic inequalities

e.g solve $x^2 + 5x - 24 \ge 0$

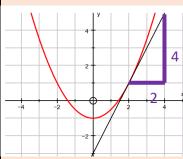
- 1. Factorise: $(x + 8)(x 3) \ge 0$
- 2. Solve: x = -8, x = 3
- 3. Sketch the graph
- 4. Values that satisfy the inequality $x \le -8$, $x \ge 3$

Graphs that need to be recognised:





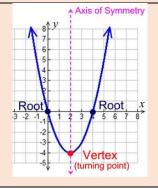
Gradients of curves



between these two values of x.

Gradient of a curve at a point = gradient of the tangent at the point

Turning point and roots of a quadratic equation



Equation of a circle centre (0, 0) $x^2 + y^2 = r^2$

Functions

f(4): Substitute 4 into the function

f(g(x)): Substitute g(x) into f(x) i.e. replace all values of x in f(x) with the entire function g(x)

e.g.
$$f(x) = 2x + 3$$
, $g(x) = x - 3$, $fg(x) = 2(x-3) + 3$

Algebriac proof – toolkit Straight line graphs **Quadratic Formula** Even numbers: y = mx + cm =____ x =Odd numbers: _____ Sum: _____ Product: _____ **Linear Inequalities** Difference: Open circle: ____ or ____ Show it's a multiple: _____ Show it's even: show it's_____ gradient gradient Closed circle: ___ or ____ Show it's odd: show it's *m* = —— = — Parallel lines – have equal gradients **Velocity / Time Graphs Completing the square** Quadratic expression factorised by completing the Gradient = _____ Perpendicular lines – square: If L_1 and L_2 are perpendicular then m_2 = $(x + a)^2 + b$ Area = _____ Turning point of graph occurs at (___, ___) Solve quadratic inequalities Graphs that need to be recognised: sketch e.g solve $x^2 + 5x - 24 \ge 0$ Exponential Quadratic Reciprocal 1. 0 1 2 3 4 5 6 7 8 9 10 2. Iteration – showing a root lies between 2 points: If there is ______ for y for two particular values 3. of x then we can say there is a _____ between these Direct Proportion **Inverse Proportion** values of x and we can say that the equation f(x) = 0 will have 4. a solution between these two values of x. Label the turning point, **Gradients of curves** Gradient of a curve at a roots and axis of point = _____ symmetry of the Equation of a circle centre (0, 0) is ___ quadratic graph **Functions** *f*(4): _____ f(g(x)): _______. i.e. replace all values of _____ in _____ with the **entire**

function

e.g. f(x) = 2x + 3, g(x) = x - 3, fg(x) =

Geometry and measure - Higher

Trigonometry

$$S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$$

Example – finding a side:

$$\sin 37 = \frac{x}{5}$$

 $x = 5 \times \sin 37^{\circ}$



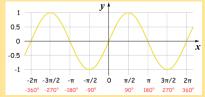
Example – finding a side:

$$\tan y = \frac{3.2}{7.1}$$

$$y = tan^{-1} \left(\frac{3.2}{7.1} \right).$$



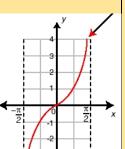
Sine Curve



Cosine Curve



Tangent Curve



Angles in parallel lines







Corresponding angles are equal

Alternate angles are equal

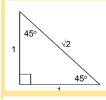
Co-interior angles are equal

Volume & surface area

Learn the cylinder

Exact Trig values

Angle (θ)	sin(θ)	cos(θ)	tan(θ)
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined





Simple vector notation

a: movement along the x-axis (left or right)

b: movement along the y-axis (up or down)

-a: movement left

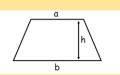
 $\binom{2}{6} + \binom{7}{-3} = \binom{9}{3}$

Operations with vectors

-b: movement down

Area of a trapezium

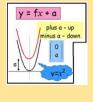
$$A = \frac{1}{2}(a+b)h$$

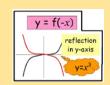


 $V = \pi r^2 h$

 $SA = 2\pi r^2 + \pi dl$

Transformation of a graph





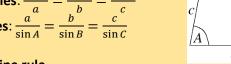


Sine rule

angles:
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

sides: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

If $b = \binom{4}{-2}$, then $3b = \binom{12}{-6}$



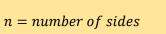
Cosine rule

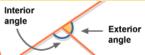
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of a triangle

$$\frac{1}{2}ab\sin C$$

Angles in regular polygons



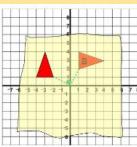


Interior angle + exterior angle = 180°

Exterior angle =
$$\frac{360}{n}$$

 $n = \frac{360}{Exterior \ angle}$

Transformations - rotation - describing:



Always use tracing paper. Describe:

- 1. It's a rotation
- 2. Size of rotation in degrees
- 3. Orientations: clockwise or anticlockwise
- 4. Centre of rotation given as a coordinate (x,y)

Transformation - translation

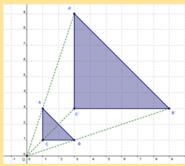
Vector $\binom{6}{-4}$ 6 right, 4 down

2. The scale factor (if the image is smaller than the object the scale factor is fractional e.g. ½)

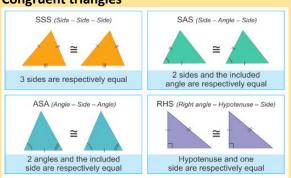
Transformations - enlargement - describing:

1. It's an enlargement

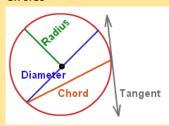
3. The centre of enlargement given as a coordinate



Congruent triangles



Circles



$$Area = \pi r^2$$

 $Circumference = \pi d$

Sector
$$Area = \frac{\theta}{360}\pi r^2$$

Arc length $= \frac{\theta}{360}\pi d$

Pythagoras' Theorem



Only applies to right angled triangles.

Can be used to find the height of an isosceles triangle



Can be used to find the length distance perween two coordinates

Circle Theorems



Angle at the centre is twice the angle at the circumference



Angles in a semicircle are 90°.



Angles in the same segment are egual.



Opposite angles of a cyclic quadrilateral add up to 180).



Alternate segment theorem.



Tangents from an external point are equal in length.

The tangent to a circle is perpendicular (90°) to the radius

Similar shapes

Same shape, different sides The ratio of the lengths of corresponding sides are equal

Length scale factor = x

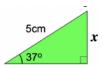
Area scale factor = x^2

Volume scale factor = x^3

Trigonometry

Fill the blanks: S - C - T -

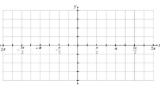
Show how to find x:



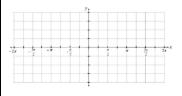
Show how to find y:



Sine Curve



Cosine Curve



Tangent Curve



Angles in parallel lines





Corresponding angles are _____

Alternate angles are

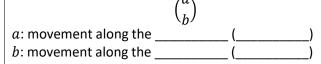
Co-interior angles are _____

Exact Trig values

y=fx + a

	0°	30°	45°	60°	90°	1 450 \(\sqrt{2}\)
$\sin \theta$						45°
$\cos\theta$						1
$tan\theta$						\wedge
						2 /30°

Simple vector notation

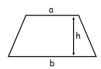


-a: movement -b: movement

Volume & surface area

Learn the cylinder

Area of a trapezium
$$A =$$



V =

SA =

Operations with vectors

$$\binom{2}{6}+\binom{7}{-3}=\binom{9}{2} \text{ if } b=\binom{4}{-2} \text{, then } 3b=\binom{9}{2}$$

Transformation of a graph: sketch

y=f(-x)

y=f(x-a)

Write down:

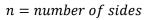
Sine rule angles: sides:

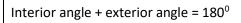
y=-fx

Cosine rule

Area of a triangle

Angles in <u>regular</u> polygons





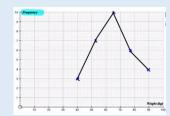
Exterior angle = —

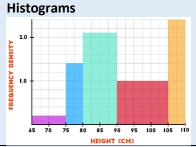
Transformations – rotation – describing: Transformations – enlargement - describing: **Circle Theorems** 1. Always use tracing paper. 2. 3. Describe: 1. Transformation - translation Vector $\binom{6}{-4}$ 6 ______, 4 _____ **Congruent triangles Circles** Draw your SSS (Side - Side - Side) SAS (Side - Angle - Side) own arrow to label an arc on the diagram 2 sides and the included 3 sides are respectively equal angle are respectively equal ASA (Angle - Side - Angle) RHS (Right angle - Hypotenuse - Side) Sector Area =Area =Hypotenuse and one side are respectively equal 2 angles and the included side are respectively equal Circumference = Arc length = Similar shapes Pythagoras' Theorem Same shape, different sides The ratio of the lengths of corresponding sides are Only applies to ______ triangles. equal Length scale factor = Can be used to find the height of an ____ triangle Area scale factor = Can be used to find the length distance between two Volume scale factor =

Frequency Polygons

- 1. Plot frequency at the mid-point
- 2. Join with straight lines

Weight w (kg)	Frequency
30 ≤ w < 50	3
50 ≤ w < 55	7
55 ≤ w < 75	10
75 ≤ w < 80	6
80 ≤ w < 100	4

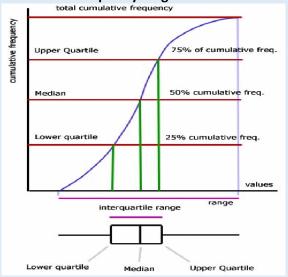




FD = Frequency density

$$FD = \frac{Frequency}{Class\ Width}$$

Cumulative Frequency Diagrams and Box Plots



Averages from a frequency table

Mean: $\frac{\sum fw}{\sum f}$; where, w is the midpoint of the group.

Median group: find which group the $\frac{n+1}{2}th$, value lies. Where, n is the total frequency.

E.G. in this table 51.5th value which lies in group $8 < w \le 12$ (using the cumulative frequency

Weight of box (w kg)	Frequency
0 < w ≤ 4	11
4 < w ≤ 8	16
8 < w ≤ 12	29
12 < w ≤ 16	26
16 < w ≤ 20	20

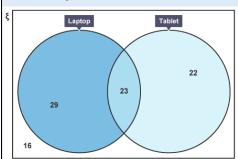
Venn Diagrams

Notation

A - all elements in A

A' – all elements **not in A** B – all elements **in B**

B' – all elements not in B



A UB – all the elements in A or B or both

A \cap B – all the elements in both A and B

Information given: 90 pupils were surveyed 52 said they owned a laptop.

45 said they owned a tablet.

23 said they owned both.

Expected outcomes

Relative frequency: $frequency \div total \ trials$

Expected outcome = probability x number of trials

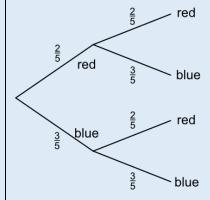
E.g. A biased spinner is spun 800 times. The probabilities is lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

Result	Red	Green	Brown	Yellow
Probability		0.48	0.2	

$$P(Y) = (1 - 0.48 - 0.2) \div 2 = 0.32 \div 2 = 0.16$$

Expected yellow = $0.16 \times 800 = 128$

Tree diagrams



Multiply along the branches to find each probability.

- 1. Probability that a red counter is picked both times $P(RR) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$
- 2. Probability that the counters are different colours = P(RB) + P(BR) = $\frac{2}{5}$ x $\frac{3}{5}$ + $\frac{3}{5}$ x $\frac{2}{5}$ = $\frac{12}{25}$

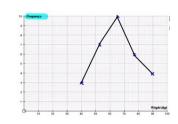
Probability and Statistics - Higher

Frequency Polygons

1.

2.

Weight w (kg)	Frequency
30 ≤ w < 50	3
50 ≤ w < 55	7
55 ≤ w < 75	10
75 ≤ w < 80	6
80 ≤ w < 100	4

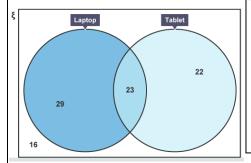


Histograms 2.0 4.5 70 75 80 85 90 95 100 105 110 HEIGHT (CH)

FD = Frequency density

$$FD =$$

Venn Diagrams



Information given:
____ pupils were
surveyed
____ said they
owned a laptop.
___ said they
owned a tablet.
___ said they
owned both.

Notation

A –

A' -

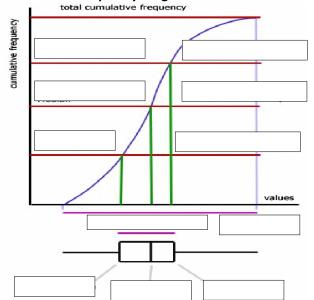
B -

B' -

AUB-

A∩B-

Cumulative Frequency Diagrams and Box Plots



Averages from a frequency table

Mean: $\frac{\sum fw}{\sum f}$; where, w is the ______ of the group.

Median group: find which group the $\frac{n+1}{2}th$, value lies. Where, n is the total frequency.

E.G. in this table 51.5th value which lies in _____ (using the cumulative frequency)

Weight of box (w kg)	Frequency
0 < w ≤ 4	11
4 < w ≤ 8	16
$8 \le w \le 12$	29
12 < w ≤ 16	26
16 < w ≤ 20	20

Expected outcomes

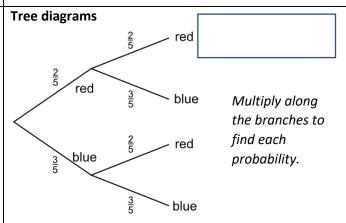
Expected outcome = _____x number of _____

E.g. A biased spinner is spun 800 times. The probabilities is lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

Result	Red	Green	Brown	Yellow
Probability		0.48	0.2	

$$P(Y) =$$

Expected yellow =



- 3. Probability that a red counter is picked both times P(RR) =
- 4. Probability that the counters are different colours =