Maths – KS4 Foundation

Fact Sheets:

- Number, Ratio and Proportion
- Algebra
- Geometry and Measures
- Probability and statistics

Number Ratio and Proportion - Foundation

Estimate
Round each value to one significant figure

Standard form
$a \times 10^n$, where $1 \leq a < 10$

Reciprocal
Reciprocal of 7 is $\frac{1}{7}$, reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ etc

Sequences
Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21
Geometric Sequence: each term is multiplied but he same constant to get the next number.
E.g. 3, 12, 48, 191, …. (x by 4 each time)

Squares and Cubes
Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225 etc

Cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, etc

Sharing in a given Ratio

A Add the ratio parts
D Divide the amount by the total parts
A and
M Multiply the ratio by the value of one part

e.g. share £420 in the ratio 2:5
2 + 5 = 7
420 ÷ 7 = £60
2: 5
(x60) (x60)
£120 : £300

Simplifying Ratio
Divide both sides by the highest common factor

Simplifying Ratio 1:n
Divide both sides by the highest factor of the left hand side

2m: 180cm
200cm: 180cm
2:1.8
1: 0.9

Fractions

Add and Subtract – ensure the fractions have the same denominator before adding numerators

$$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$

Multiply – multiply numerators and denominators

$$\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

Divide – take reciprocal of the second fraction and then multiply the new numerators and denominators

$$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$$

Percentages

Finding percentages of an amount

1%	÷100
5%	÷20
20%	÷5
25%	÷4
50%	÷2

Multipliers:
To find the multiplier for a percentage, divide by 100

Use multipliers on a calculator paper
e.g. 35% of 370 = 0.35 x 370

Increasing and decreasing a given amount
Calculator:

$$Orginal\ Amount\ x\ mutiplier = new\ amount$$

Non-calculator: find the increase or decrease and add to the original amount

Finding percentage increase or decrease (profit/loss)

$$\frac{value\ of\ increase/decrease}{Original} \times 100$$

Writing an amount as a percentage of the original

$$\frac{Amount}{Original} \times 100$$

Reverse Percentage – finding the original amount

$$Orginal\ Amount = \frac{New\ Amount}{multiplier}$$

Growth & Decay / Compound interest

$$original\ amount \times multiplier^{time}$$

Where the multiplier is the percentage, increase or decrease from 100%, converted to a decimal.
e.g.
30% decrease is 70% = 0.7
30% increase is 130% = 1.3

Dividing by decimals:

1. Write the calculation as a fraction
2. Form an equivalent fraction to makes integers (multiply by powers of 10)
3. Use short division (bus stop) to calculate

e.g. $460 \div 0.4 = \frac{460}{0.4} = \frac{4600}{4} = 1150$

Conversions

10 millimetres = 1 centimetre	15 minutes = 0.25 hours
100 centimetres = 1 metre	30 minutes = 0.5 hours
1000 metres = 1 kilometre	45 minutes = 0.75 hours
$1000cm^3$ = 1 litre	1000g = 1 kilogram
1000ml = 1 litre	1000kg = 1 tonne

Compound Units (rearrange as necessary)

$$Speed = \frac{Distance}{Time}$$

$$Area = \frac{Force}{Pressure}$$

$$Density = \frac{Mass}{Volume}$$

Error Intervals

least possible value $\leq x <$ greatest possible value

e.g. A fence is 30 m long to the nearest 10 m.

$$25\text{ m} \leq l < 35\text{ m}$$

Truncation

Truncation is a method of approximating a decimal number by dropping all decimal places past a certain point without rounding.

e.g. Truncate 3.14159265 to 4 decimal places.
= 3.1415

Negative numbers

<u>Adding and subtracting: (vertical number lines help)</u>
-3 – 5 = - 8
-3 + 5 = 2
-3 - - 5 = -3 + 5 = 2
-3 - + 5 = -3 – 5 = -8
-3 + - 5 = -3 – 5 = -8

<u>Multiplying and dividing:</u>
Different signs – answer will be negative
+ x - = -, - x + = -
Same signs – answer will be positive
- x - = +

Ordering fractions

Calc: use division to write each fraction as a decimal
Non-calc: write fractions with common denominators

Order of operations

Bracket
Indices
Division and **M**ultiplication
Addition and **S**ubtraction

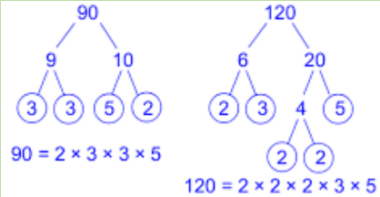
Rounding to significant figures

Start from the first **non-zero** number and round as normal, but ensure the place value is correct
e.g. 345,635 to 2SF = 350,000
0.0060821 to 3SF = 0.0608

Index Laws

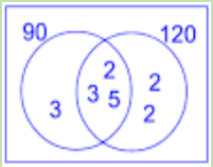
$$a^n \times a^m = a^{n+m}$$
$$a^n \div a^m = a^{n-m}$$
$$(a^n)^m = a^{nm}$$
$$a^0 = 1$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Prime Factorisation



HCF and LCM of 90 and 120 <u>(Factor Tree & Venn Diagram)</u>

HCF is the product of common factors
LCM is the product of common factors and remaining factors.



HCF: 2x3x5
LCM: 2^3x3^2x5

Number Ratio and Proportion - Foundation

Estimate
Round each value to _____

Standard form
$a \times$ _____^{n}, where $1 \leq a < 10$

Reciprocal
Reciprocal of 7 is _____, reciprocal of $\frac{2}{3}$ is ___ etc

Sequences
Fibonacci sequence: _____
Geometric Sequence: _____
_____
E.g. 3, 12, 48, 191, …. (x by 4 each time)

Squares and Cubes
Square numbers: _____
_____

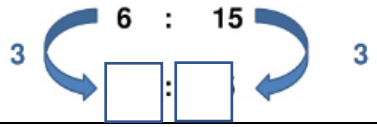
Cube numbers: _____
_____

Sharing in a given Ratio

A	Add the ratio parts
D	Divide the amount by the total parts
A	and
M	Multiply the ratio by the value of one part

e.g. share £420 in the ratio 2:5

Simplifying Ratio
Divide both sides by the highest common factor



Simplifying Ratio 1:n
Divide both sides by the highest factor of the left hand side
Simplify: 2m: 180cm

Fractions

Add and Subtract – ensure the fractions have the same _____ before adding the _____

$$\frac{4}{5} - \frac{1}{3} =$$

Multiply – multiply _____ and _____

$$\frac{4}{5} \times \frac{1}{3} =$$

Divide – take _____of the second fraction and then _____the new _____and _____

$$\frac{4}{5} \div \frac{1}{3} =$$

Percentages

Finding percentages of an amount

1%	÷_____
5%	÷_____
20%	÷_____
25%	÷_____
50%	÷_____

Multipliers:
To find the multiplier for a percentage, divide by 100

Use multipliers on a calculator paper
e.g. 35% of 370 = 0.35 x 370

Increasing and decreasing a given amount
Calculator:

$$\_\_\_\_\_ = new\ amount$$

Non-calculator: find the increase or decrease and add to the original amount

Finding percentage increase or decrease (profit/loss)

$$\frac{\_\_\_\_\_}{Original} \times 100$$

Writing an amount as a percentage of the original

$$\frac{Amount}{\_\_\_\_\_} \times \_\_\_\_\_$$

Reverse Percentage – finding the original amount

$$Orginal\ Amount = \_\_\_\_\_$$

Growth & Decay / Compound interest

_____ × _____

Where the multiplier is the percentage, increase or decrease from 100%, converted to a decimal.
e.g.
30% decrease is 70% = _____
30% increase is 130% = _____

Dividing by decimals:

1.
2.
3.

e.g. 460 ÷ 0.4 =

Conversions

10 millimetres = _____
100 centimetres = _____
30 minutes = _____ hours
1000 metres = _____
45 minutes = _____ hours
1000cm^3 = _____ 1000g = _____
1000ml = _____ 1000kg = _____

Compound Units (rearrange as necessary)

$Speed =$ _____

$Area =$ _____

$Density =$ _____

Error Intervals

least possible value $\leq x <$ greatest possible value

e.g. A fence is 30 m long to the nearest 10 m.

_____ $\leq l <$ _____

Truncation

Truncation is _____
_____
_____

e.g. Truncate 3.14159265 to 4 decimal places.
= _____

Negative numbers

<u>Adding and subtracting: (vertical number lines help)</u>
-3 – 5 =
-3 + 5 =
-3 - - 5 =
-3 - + 5 =
-3 + - 5 =

<u>Multiplying and dividing:</u>
Different signs – answer will be _____
+ x - = _____, - x + = _____
Same signs – answer will be _____
- x - = _____

Ordering fractions

Calc: use division to write each fraction as a decimal
Non-calc: write fractions with common denominators

Order of operations

B_____
I_____
D_____ and **M**_____
A_____ and **S**_____

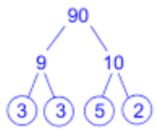
Rounding to significant figures

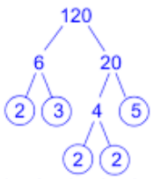
Start from the first _____ number and round as normal, but ensure the place value is correct
e.g. 345,635 to 2SF =
0.0060821 to 3SF =

Index Laws

$a^n \times a^m =$
$a^n \div a^m =$
$(a^n)^m =$
$a^0 =$
$a^{-n} =$
$a^{\frac{n}{m}} =$

Prime Factorisation

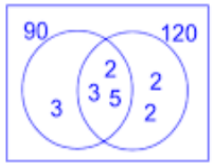




=_____ = _____

HCF and LCM of 90 and 120 <u>(Factor Tree & Venn Diagram)</u>

HCF is the _____
LCM is the _____



HCF: _____

LCM: _____

Notation

ab = a x b

a^2 = a x a

$(2a)^3$ = 2a x 2a x 2a

$(a + b)^2$ = (a + b)(a + b)

Definitions

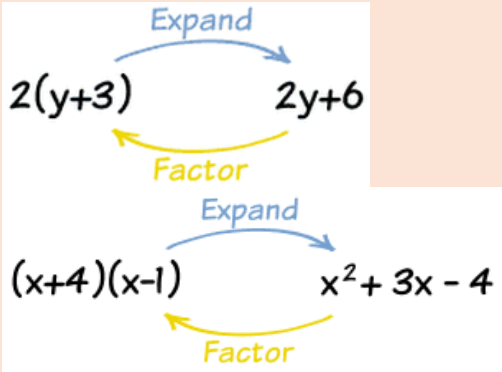
Expression – no equal signs e.g. 2x + 3, 2y, $(3x - 2)^2$

Equations – equal signs, can be solved, e.g. y + 4 = 10

Identities – identical/equivalent to e.g. 2(y + 4) ≡ 2y + 8

Formulae – equal signs, more than one unknown e.g. A= ½bh

Factorising and expanding

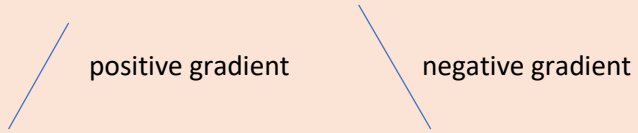


Straight line graphs

$y = mx + c$

$m = gradient$

$c = y - intercept$



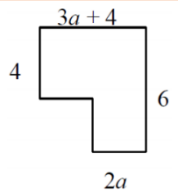
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x}$$

Parallel lines – have equal gradients

Simplifying expressions by collecting like terms

Always circle the sign IN FRONT of the term to avoid errors.

$$3x - 7b - x + 9b \equiv 2x + 2b$$



Typical Exam Q: Create an expression for the perimeter of the shape by adding and collecting like terms.

If the perimeter is given as 20cm, for example, you can create an equation:

$$4 + 3a + 4 + 6 + 2a = 20$$

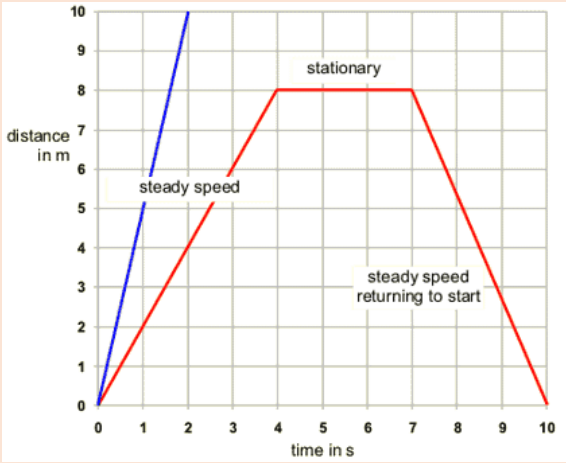
$$5a + 14 = 20$$

Simplifying expressions multiplication and division

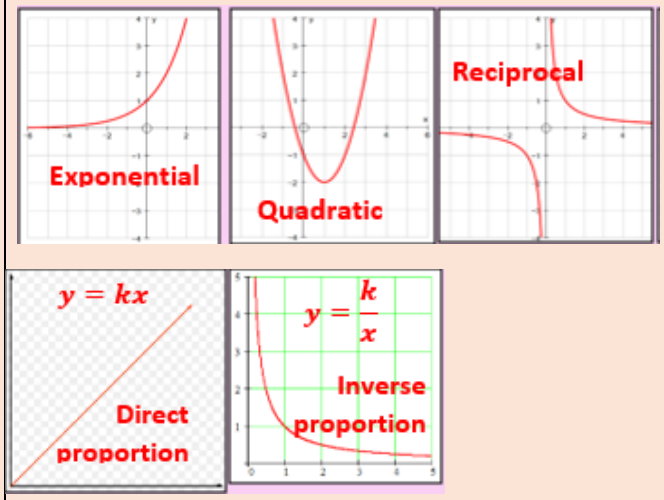
$2ma^2 \times 7ma = 14m^2a^3$

$\frac{18b^6}{3ab^2} = \frac{6b^4}{a}$

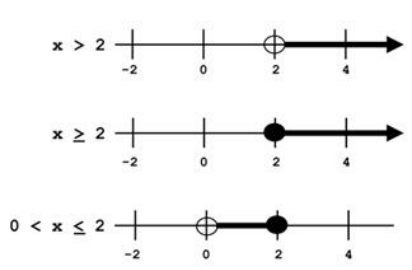
Distance / Time Graphs



Graphs that need to be recognised



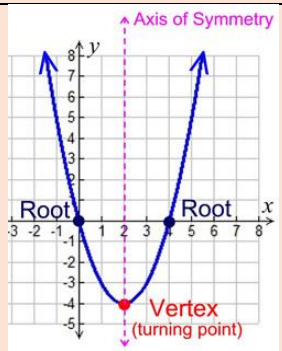
Inequalities



Open circle: < / >

Closed circle: ≤ / ≥

Turning point and roots of a quadratic equation



Finding the nth term of a linear sequence

5, 7, 9, 11, 13, ….

1. Find the common difference: 2
2. This is the coefficient of n: 2n
3. Find the difference between the coefficient of n and the first term 5 – 2 = 3
4. Add this to the amount of n

   2n + 3

Notation

ab =

a^2 =

$(2a)^3$ =

$(a + b)^2$ =

Definitions

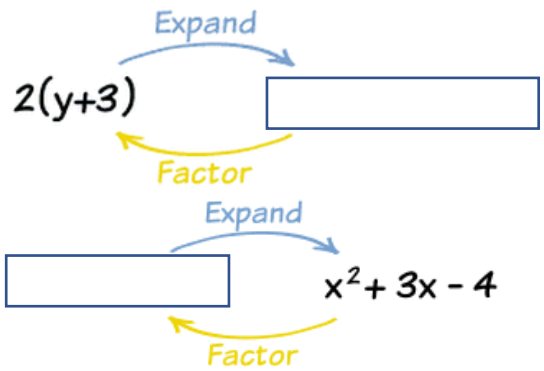
Expression –

Equations –

Identities –

Formulae –

Factorising and expanding



Straight line graphs

$y = mx + c$

$m =$ _____

$c =$ _____

_____ gradient

_____ gradient

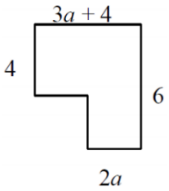
$m =$ _____ $=$ _____

Parallel lines – _____

Simplifying expressions by collecting like terms

Always circle the sign IN FRONT of the term to avoid errors.

$$3x - 7b - x + 9b \equiv 2x + 2b$$

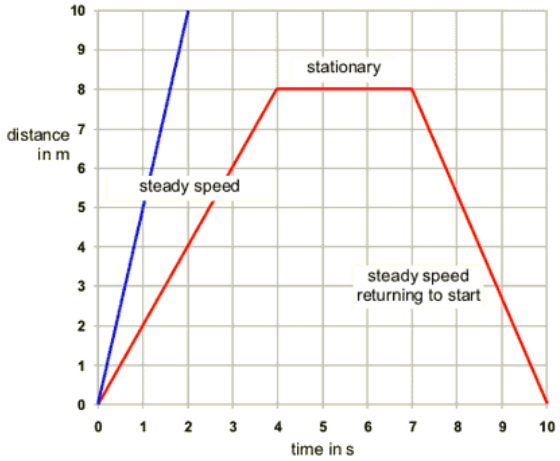


Typical Exam Q: Create an expression for the perimeter of the shape by adding and collecting like terms.

If the perimeter is given as 20cm, for example, you can create an equation:

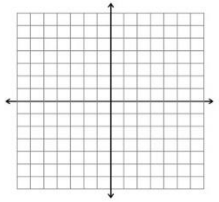
ANSWER =

Distance / Time Graphs

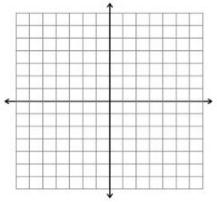


Graphs that need to be recognised: sketch

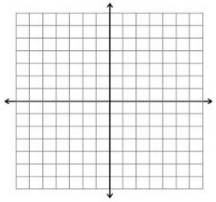
Exponential



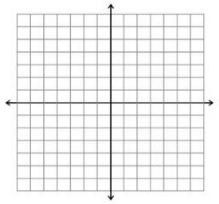
Quadratic



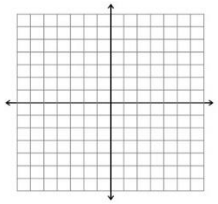
Reciprocal



Direct Proportion



Inverse Proportion

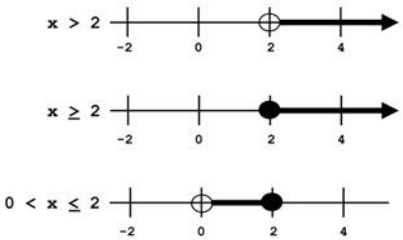


Simplifying expressions multiplication and division

$2ma^2 \times 7ma =$ _____

$\frac{18b^6}{3ab^2} =$ _____

Inequalities



Open circle: _____

Closed circle: _____

Label the turning point, roots and axis of symmetry of the quadratic graph

Show how to find the nth term of a linear sequence

5, 7, 9, 11, 13, ….

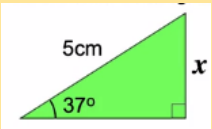
Geometry and Measures - Foundation

Trigonometry

$$S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$$

Example – finding a side:

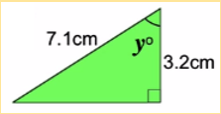
$\sin 37 = \frac{x}{5}$



x = 5 x sin 37^0

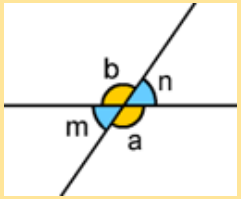
Example – finding a side:

$\tan y = \frac{3.2}{7.1}$

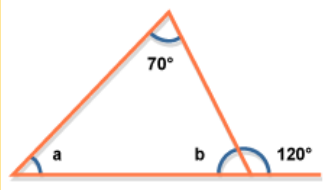


$y = tan^{-1}\left(\frac{3.2}{7.1}\right)$.

Angle Facts



Vertically opposite angles are equal: a=b and m=n

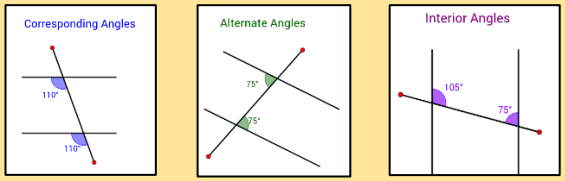


Angles in a triangle sum to 180^0.

Angles on a straight line sum to 180^0.

E.G: b=60^0 so a = 50^0

Angles in parallel lines



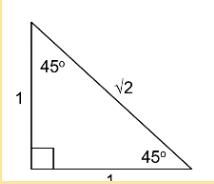
Corresponding angles are equal

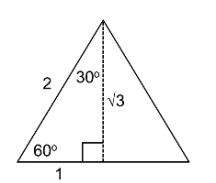
Alternate angles are equal

Co-interior angles are equal

Exact Trig values

Angle (θ)	sin(θ)	cos(θ)	tan(θ)
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined





Simple vector notation

$$\binom{a}{b}$$

a: movement along the x-axis (left or right)
b: movement along the y-axis (up or down)

$-a$: movement left $\qquad -b$: movement down

Operations with vectors

$\binom{2}{6} + \binom{7}{-3} = \binom{9}{3}$

If $b = \binom{4}{-2}$, then $3b = \binom{12}{-6}$

Volume & surface area

Volume = area of cross section x length

Surface area = area of all the faces of a 3D shape

Learn the cylinder

$$V = \pi r^2 h$$
$$SA = 2\pi r^2 + \pi dl$$

Types of triangles

Right angled
Isosceles
Equilateral
Scalene

Types of quadrilaterals

Square
Rectangle
Parallelogram
Rhombus
Trapezium
Kite

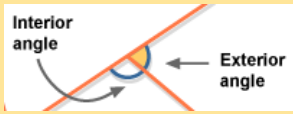
Area of key shapes

Triangle : A = $\frac{b \times h}{2}$ (h = perpendicular height)

Parallelogram: A = b x h (h = perpendicular height)

Trapezium: A = $\left(\frac{a+b}{2}\right)$ x h (add together the parallel sides, divide the total by 2, and then multiply by the perpendicular height between the parallel sides)

Angles in regular polygons



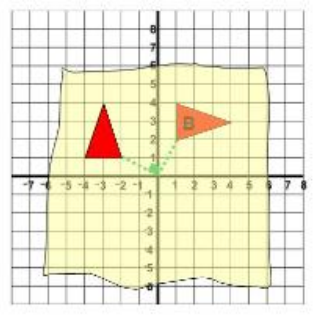
$n = number\ of\ sides$

Interior angle + exterior angle = 180^0

Exterior angle = $\frac{360}{n}$

$$n = \frac{360}{Exterior\ angle}$$

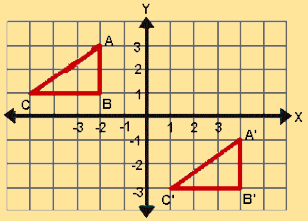
Transformations – rotation



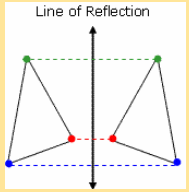
Always use tracing paper.
Describe:

1. It's a rotation
2. Size of rotation in degrees
3. Orientations: clockwise or anticlockwise
4. Centre of rotation given as a coordinate (x,y)

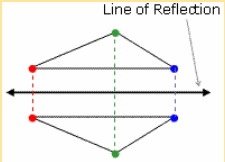
Transformations – translations and reflections



Translate triangle ABC to A'B'C' with the vector $\binom{6}{-4}$

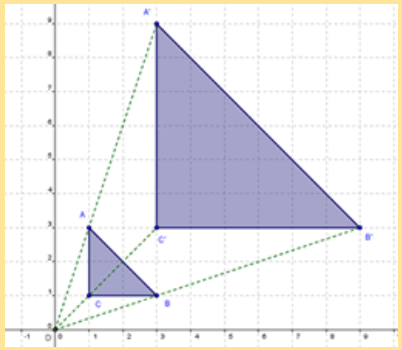


Reflection in the line x=a



Reflection in the line y=a

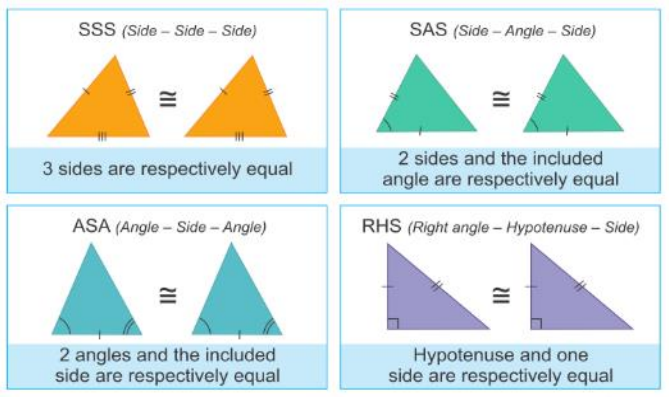
Transformations - enlargement



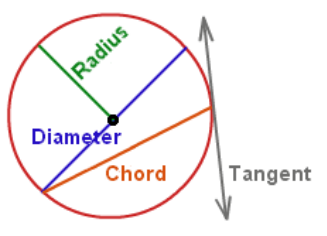
Describe:

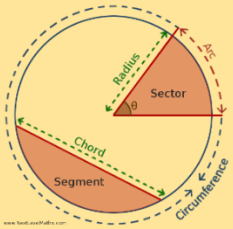
1. It's an enlargement
2. The scale factor (if the image is smaller than the object the scale factor is fractional e.g. ½)
3. The centre of enlargement given as a coordinate

Congruent triangles



Circles





$Area = \pi r^2$

Sector $Area = \frac{\theta}{360}\pi r^2$

$Circumference = \pi d$

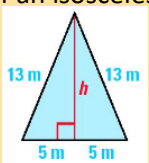
Arc length = $\frac{\theta}{360}\pi d$

Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

Only applies to right angled triangles.

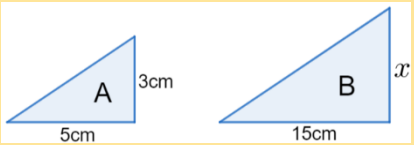
Can be used to find the height of an isosceles triangle



Can be used to find the length distance between two coordinates

Similar shapes

Same shape, different sides
The ratio of the lengths of corresponding sides are equal



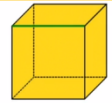
Length scale factor = 15 ÷ 5 = 3

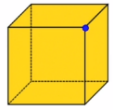
x = 3cm x 3

3D notation

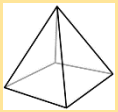
<u>Cube:</u>

Faces: 6 Edges: 12 Vertices: 8





<u>Square based pyramid:</u>

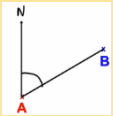


F = 5, E = 8, V = 5

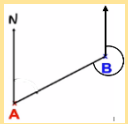
Bearings

Measure from the North
Measured in a clockwise direction
Written using 3 digits

Bearing of B **<u>from A</u>** (start at A)



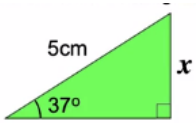
Bearing of A **<u>from B</u>** (start at B)



Trigonometry

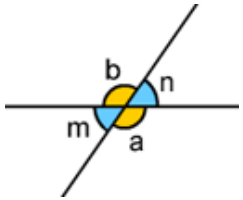
Fill the blanks: $S - C - T -$

Show how to find x:

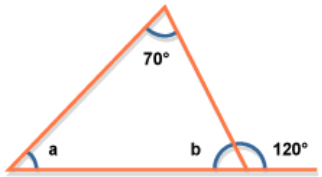


Show how to find y:

Angle Facts



Vertically opposite angles are _____

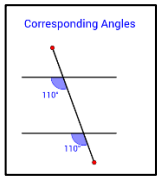


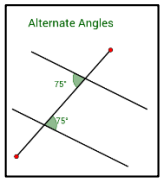
Angles in a triangle sum to _____.

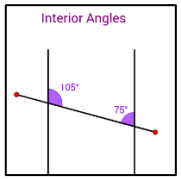
Angles on a straight line sum to _____.

E.G: b=_____ so a = _____

Angles in parallel lines







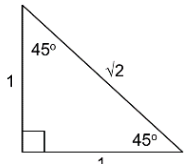
Corresponding angles are _____

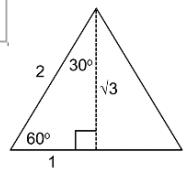
Alternate angles are _____

Co-interior angles are _____

Exact Trig values

	0°	30°	45°	60°	90°
sinθ					
cosθ					
tanθ					





Simple vector notation

$\binom{a}{b}$

a: movement along the _____ (_____)

b: movement along the _____ (_____)

$-a$: movement _____ $-b$: movement _____

Operations with vectors

$\binom{2}{6} + \binom{7}{-3} = (\ \ )$ If $b = \binom{4}{-2}$, then $3b = (\ \ )$

Volume & surface area

Volume = _____ x _____

Surface area = area of _____

Learn the cylinder

$V =$ _____

$SA =$ _____

Types of triangles

_____

_____

_____

_____

Types of quadrilaterals

_____

_____

_____

_____

_____

_____

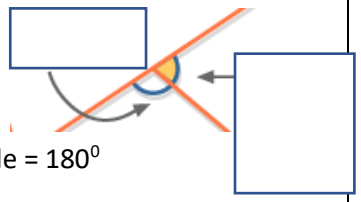
Area of key shapes

Triangle : A = ——— (h = _____ height)

Parallelogram: A = _____ (h = _____ height)

Trapezium: A = _____

Angles in regular polygons



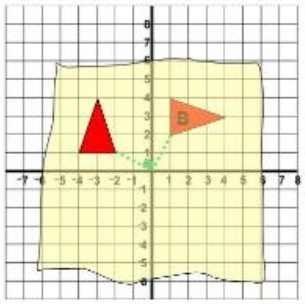
$n = number\ of\ sides$

Interior angle + exterior angle = 180^0

Exterior angle = ———

$n =$ _____

Transformations – rotation



Always use tracing paper.
Describe:

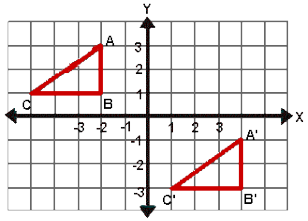
1.

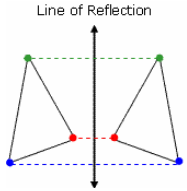
2.

3.

4.

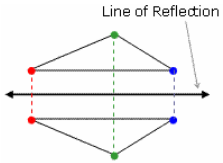
Transformations – translations and reflections





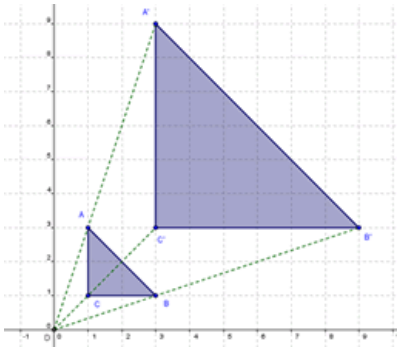
Reflection in the line _____

Translate triangle ABC to A'B'C' with the vector $\binom{6}{-4}$



Reflection in the line _____

Transformations - enlargement

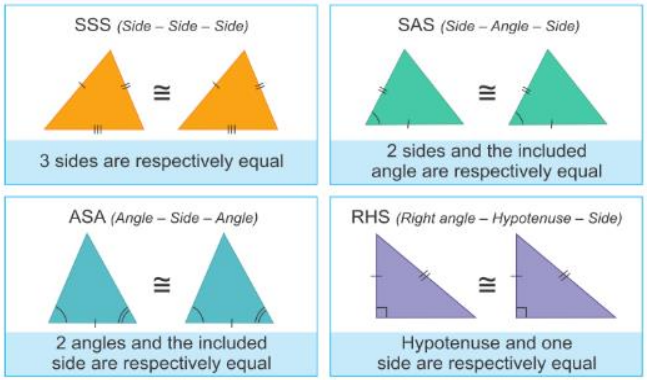


Describe:
1.

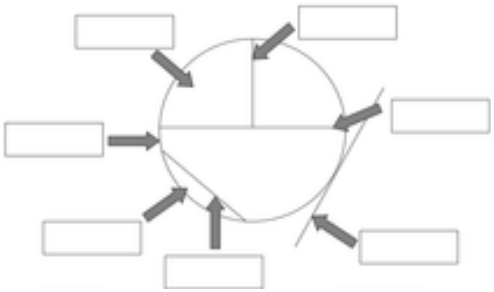
2.

3.

Congruent triangles



Circles



Draw your own arrow to label an arc on the diagram

$Area =$

Sector $Area =$

$Circumference =$

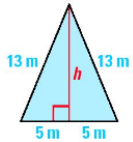
Arc length =

Pythagoras' Theorem

_____

Only applies to _____ triangles.

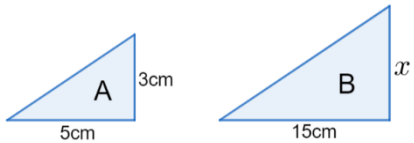
Can be used to find the height of an _____ triangle



Can be used to find the length distance between two _____

Similar shapes

Same shape, different sides
The ratio of the lengths of corresponding sides are equal



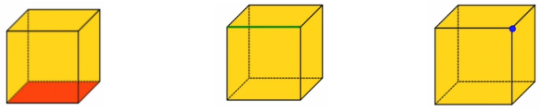
Length scale factor = _____

x = _____

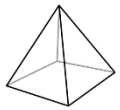
3D notation

Cube:

Faces: ____ Edges: ____ Vertices: ____



Square based pyramid:

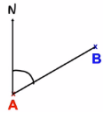


F = ____ , E = ____ , V = ____

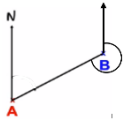
Bearings

Measure from _____
Measured in a _____ direction
Written using _____ digits

Bearing of B **from A** (start at _____)



Bearing of A **from B** (start at _____)



Probability and Statistics - Foundation

Averages

Mode: most common piece of data

Mean: Sum of the data ÷ total frequency

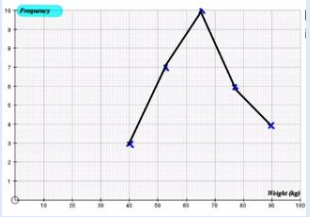
Median: order the data and find the middle value

Range: Highest value – lowest value

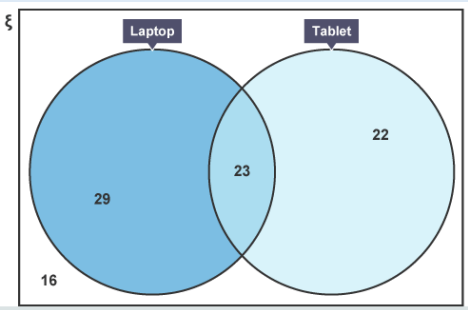
Frequency Polygons

1. Plot frequency at the mid-point
2. Join with straight lines

Weight w (kg)	Frequency
$30 \le w < 50$	3
$50 \le w < 55$	7
$55 \le w < 75$	10
$75 \le w < 80$	6
$80 \le w < 100$	4



Venn Diagrams



Information given:
90 pupils were surveyed
52 said they owned a laptop.
45 said they owned a tablet.
23 said they owned both.

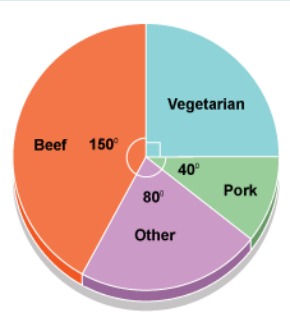
Probability Definitions

Total probability: adds to 1

Relative frequency: $frequency \div total\ trials$

Independent events: one event doesn't impact the other

Reading and Drawing Pie Charts



Find the fraction of the total

1000 people were surveyed

Beef: $\frac{150}{360} \times 1000$

Vegetarian: $\frac{90}{360} \times 1000$

Hair colour	People
Blonde	8
Brown	12
Red	3
Grey	2
Black	6

Find the fraction of the full circle.

Size of Blonde sector: $\frac{8}{31} \times 360°$

Expected outcomes

Expected outcome = probability x number of trials

E.g. A biased spinner is spun 800 times. The probabilities is lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

Result	Red	Green	Brown	Yellow
Probability		0.48	0.2	

P(Y) = (1 – 0.48 – 0.2) ÷ 2 = 0.32 ÷ 2 = 0.16

Expected yellow = 0.16 x 800 = 128

Averages from a frequency table

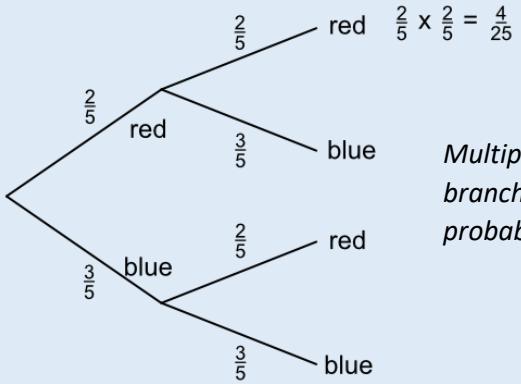
Mean: $\frac{\sum fw}{\sum f}$; where, w is the midpoint of the group.

Median group: find which group the $\frac{n+1}{2}th$, value lies. Where, n is the total frequency.

E.G. in this table 51.5th value which lies in group $8 < w \le 12$ (using the cumulative frequency

Weight of box (w kg)	Frequency
$0 < w \leqslant 4$	11
$4 < w \leqslant 8$	16
$8 < w \leqslant 12$	29
$12 < w \leqslant 16$	26
$16 < w \leqslant 20$	20

Tree diagrams



Multiply along the branches to find each probability.

1. Probability that a red counter is picked both times P(RR) = $\frac{2}{5}$ x $\frac{2}{5} = \frac{4}{25}$
2. Probability that the counters are different colours = P(RB) + P(BR) = $\frac{2}{5}$ x $\frac{3}{5} + \frac{3}{5}$ x $\frac{2}{5} = \frac{12}{25}$

Probability and Statistics - Foundation

Averages

Mode: _____

Mean: _____

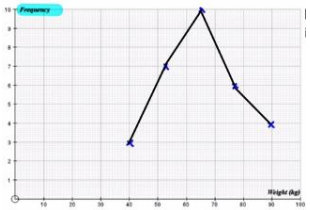
Median: _____

Range: _____

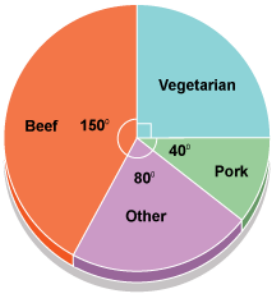
Frequency Polygons

1.
2.

Weight w (kg)	Frequency
$30 \le w < 50$	3
$50 \le w < 55$	7
$55 \le w < 75$	10
$75 \le w < 80$	6
$80 \le w < 100$	4



Reading and Drawing Pie Charts



Find the fraction of the total

1000 people were surveyed

Beef: —— × _____

Vegetarian: —— × _____

Hair colour	People
Blonde	8
Brown	12
Red	3
Grey	2
Black	6

Find the fraction of the full circle.

Size of Blonde sector: _____

Averages from a frequency table

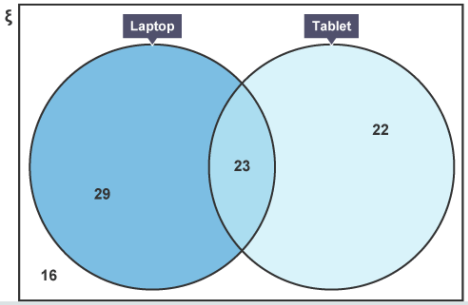
Mean: $\frac{\sum fw}{\sum f}$; where, w is the _____ of the group.

Median group: find which group the $\frac{n+1}{2}th$, value lies. Where, n is the total frequency.

E.G. in this table 51.5th value which lies in _____ (using the cumulative frequency)

Weight of box (w kg)	Frequency
$0 < w \leqslant 4$	11
$4 < w \leqslant 8$	16
$8 < w \leqslant 12$	29
$12 < w \leqslant 16$	26
$16 < w \leqslant 20$	20

Venn Diagrams



Information given:
____ pupils were surveyed
____ said they owned a laptop.
____ said they owned a tablet.
____ said they owned both.

Probability Definitions

Total probability: adds to ____

Relative frequency: _____ ÷ _____

Independent events: one event _____ impact the other

Expected outcomes

Expected outcome = _____ x number of _____

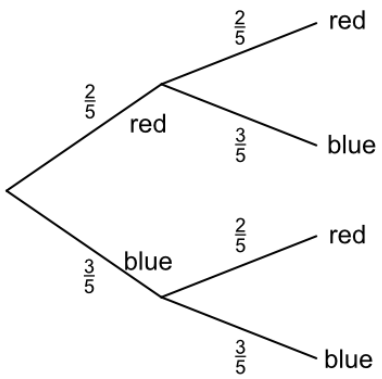
E.g. A biased spinner is spun 800 times. The probabilities is lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

Result	Red	Green	Brown	Yellow
Probability		0.48	0.2	

P(Y) =

Expected yellow =

Tree diagrams



_____ *along the branches to find each probability.*

1. Probability that a red counter is picked both times P(RR) =
2. Probability that the counters are different colours =